Correspondence

VQ-Adaptive Block Transform Coding of Images

Hakan Çaglar, Sinan Güntürk, Bülent Sankur, and Emin Anarım

***Abstract*— Two new design techniques for adaptive orthogonal block transforms based on vector quantization (VQ) codebooks are presented. Both techniques start from reference vectors that are adapted to the characteristics of the signal to be coded, while using different methods to create orthogonal bases. The resulting transforms represent a signal coding tool that stands between a pure VQ scheme on one extreme and signal-independent, fixed block transformation-like discrete cosine transform (DCT) on the other. The proposed technique has superior compaction performance as compared to DCT both in the rendition of details of the image and in the peak signal-to-noise ratio (PSNR) figures.**

***Index Terms*—Coding gain, orthogonal block transform, permutation, reference vector, vector quantization.**

I. Introduction

Block transforms (BT's) are attractive tools for the analysis and compression of images. The Karhunen–Loeve transform (KLT), while optimal among all unitary transforms, has the shortcoming of lacking of a fast transform algorithm. The discrete cosine transform (DCT), on the other hand, possesses the fast transform character and its performance is very close to that of KLT, at least for highly correlated first-order Markov processes [1], [2]. These advantages have made, in fact, the DCT the industry standard for the first generation image and video codecs [3].

Despite the preponderance of DCT algorithms in practical applications, the quest for new block transforms continues especially in the direction of transform codebook designs where each block transform is adapted to the local statistics of nonstationary fields [4], [5].

This study has provided further insight into the two well-known techniques, BT's and vector quantization (VQ). Our proposed coding technique can be interpreted as a bridge between these two techniques.

Since image signals are nonstationary, it is conjectured that a BT for which the basis functions adapt to the local statistics of the image will have superior compression performance. The adaptation of the transform bases is instrumented by constructing a transform book, which is similar in concept to a VQ codebook. In the transform book, each BT corresponds to some typical image structure or waveform. In each segment of the image an appropriate BT should be chosen based on some simple feature of the regional image, or simply using the mean square error criterion. However, such transform books contain many fewer (typically four to eight) transforms as compared to the number of vectors in a conventional VQ codebook. Notice that if some bit assignment rule were to single out the first row only in each block, then the proposed coder would reduce to a scheme similar to the VQ coder. On the other hand, in our VQ-based BT, the rows other than the first one can be interpreted as encoding the residual error of the VQ. Furthermore, in our scheme, the search effort is much less as compared to a conventional VQ coding, since our VQ size is minute (e.g., four to eight), i.e., which in turn implies that the cluster sizes are quite large. The loss in resolution due to much reduced VQ size is compensated by the use of the other $(N-1)$ orthogonal vectors in the BT. In this sense, our adaptive VQ-based BT aims to combine the best of the two worlds of BT's and VQ.

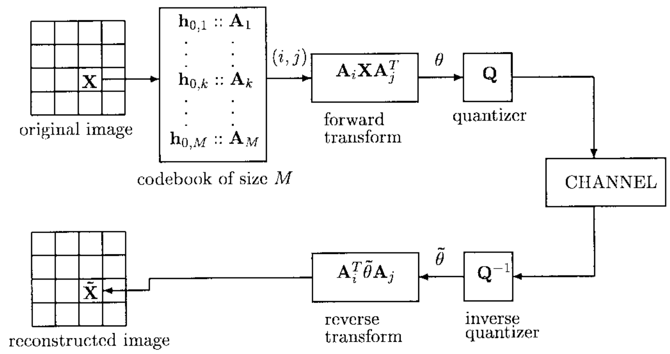

Fig. 1. Block diagram of the proposed VQ-adaptive BT technique.

In this correspondence, we intend to design "VQ-adaptive BT's," such that the basis vectors of the transform are expected to match the statistics of the input process. This can be effected by having the rows of the transform block to resemble waveform portions most typically encountered in the image class to be coded. Such an adaptive block transform can be constructed first by obtaining a reference vector that also forms, let us say, the first row, while the other rows of the transform block are generated by operations on this reference vector, as described in Sections III and IV. These two new methods that obtain an orthogonal BT from a given reference vector will be called *vector quantization-permutation based block transform (VQ-PBT)* and *vector quantization-optimization based block transform (VQ-OBT)*, while such techniques in general are referred to as VQ-adaptive block transforms from here on. Fig. 1 shows the block diagram of the proposed VQ-adaptive BT techniques.

The organization of the paper is as follows: Section II discusses a VQ-based selection of the reference vectors that will be the starting point for both the VQ-PBT and VQ-OBT types of orthogonal transform design. In Section III, we describe the new VQ-PBT type block transform design technique, while the VQ-OBT type block transform is given in Section IV. Some design examples and comparative results for both the VQ-PBT and VQ-OBT techniques are presented in Section V. Section VI gives conclusions and suggestions for future research.

II. Selection of Reference Vectors

The reference vectors in our scheme act as the seeds of the adaptive BT. Therefore, these reference vectors should be a collection of reproduction vectors and, hence, VQ techniques can be invoked to obtain them. To generate a collection of M transform blocks, M reference vectors are obtained using a VQ technique known as the Linde–Buzo–Gray (LBG) algorithm [9]. Our proposed scheme is

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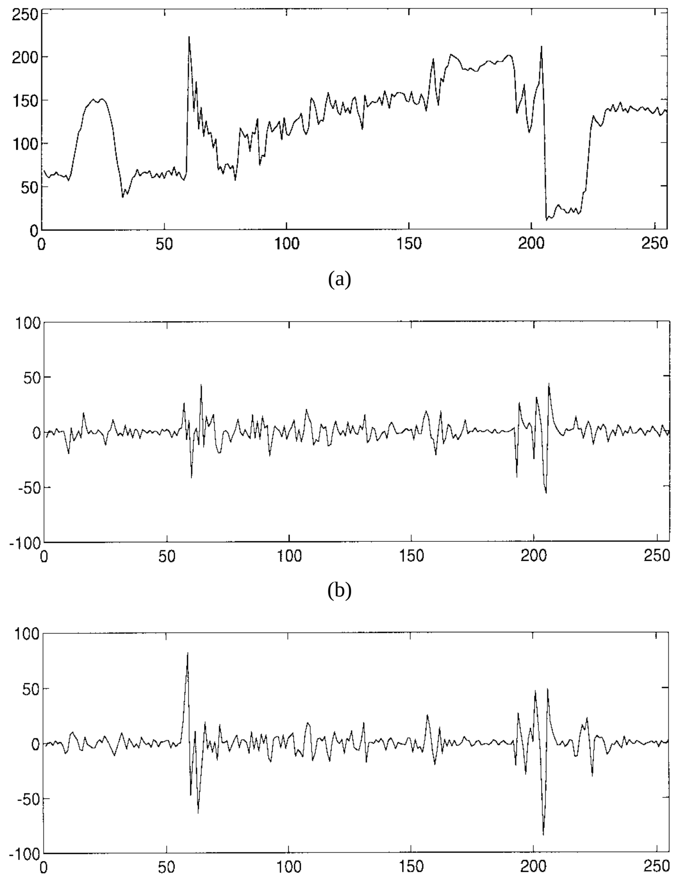


Fig. 2. (a) Sample line from Lena image. (b) Error signal for adaptive 8 × 8 VQ-PBT with transform codebook of size eight and reconstructed from two coefficients in each block. (c) Error signal for DCT with the same compression rate as above.

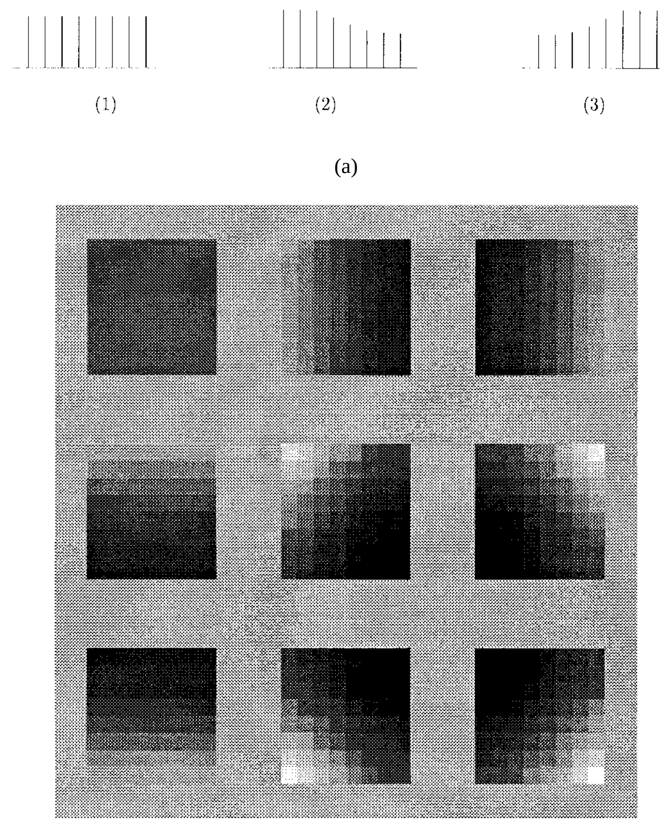


Fig. 3. (a) Reference codebook of size 3. (b) Nine image patterns formed with the outer product of the vectors in Fig. 3(a).

applicable to both one-dimensional (1-D) and two-dimensional (2-D) cases, albeit with slight differences.

1-D Case

Using a clustering technique like the LBG algorithm on the N-sized row or column segments of the images, the feature vectors $\boldsymbol{h}_m, m = 1, \cdots, M$ are obtained as the centroids of the partition. These final M reproduction vectors will be called *reference vectors* from here on. Then for each reference vector, $\boldsymbol{h}_m$, in this codebook, an orthogonal BT $\boldsymbol{B}_m$ of size $N \times N$ is generated either via signed permutation operations of this vector or an optimization in the null space of it as described in Sections III and IV. In either case these vectors remain as the first basis functions of the block transforms. Nonoverlapping segments of the signal $\boldsymbol{x}_k = [x(Nk) \cdots x(Nk + N - 1)]^T, k = 0, 1, \cdots$ are transformed to produce

$$\boldsymbol{y}_k = \boldsymbol{A}_{m_k} \boldsymbol{x}_k \tag{1}$$

where $\boldsymbol{A}_{m_k}$ is the most appropriate transform for that segment, and the transform coefficients are then quantized and coded. The overhead to index the transforms in the book is moderate since the transform book size is limited to six to seven, which necessitates three bits. However, since not all the bases are selected with equal frequency, with entropy coding the overhead reduces to approximately two bits. Thus, for example, at 1 b/pixel rate, the overhead constitutes only 3% of the bit budget.

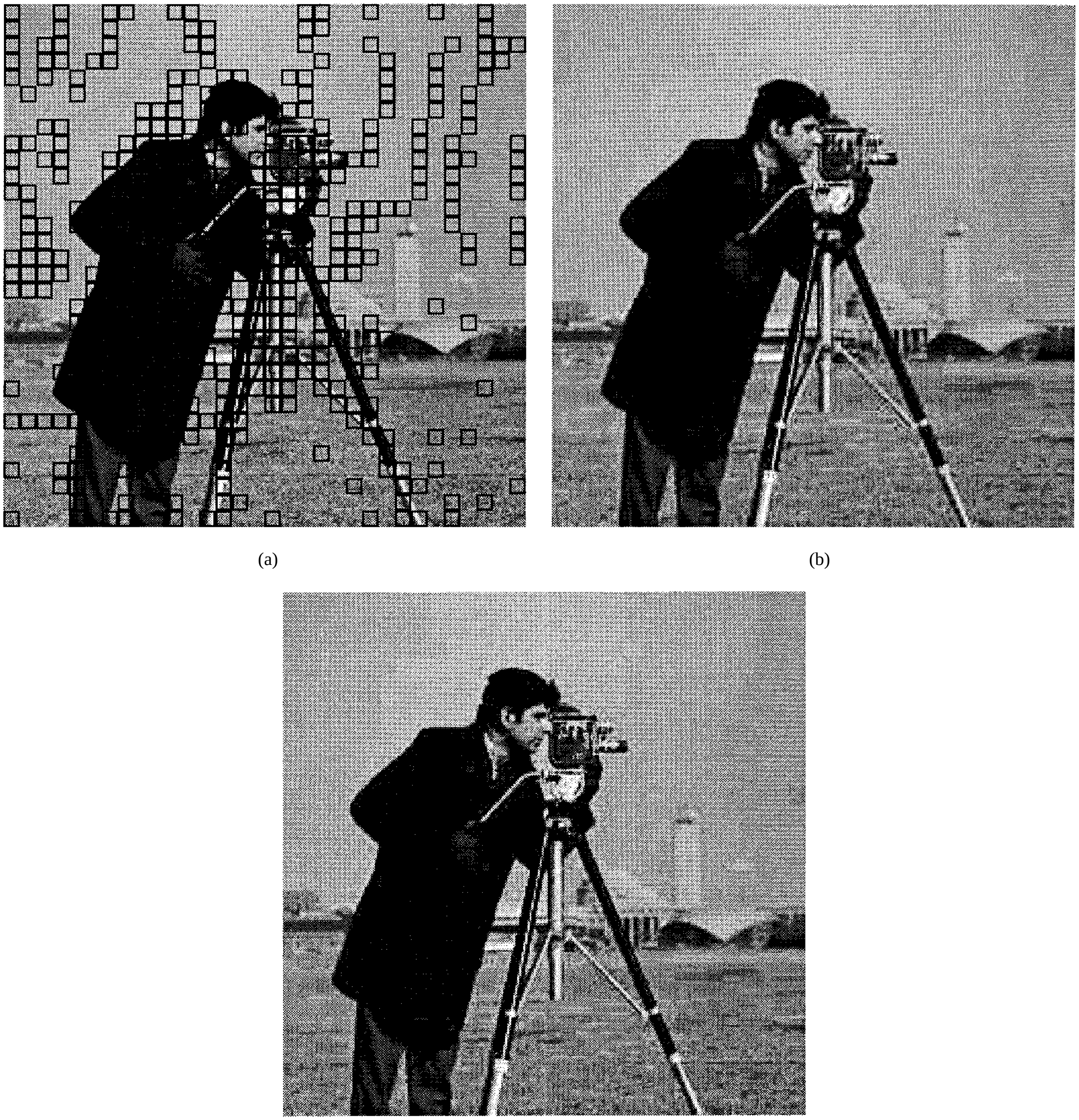
The performance of the proposed adaptive BT technique vis-a-vis DCT in the case of 1-D signals is illustrated in Fig. 2, where a sample line from the Lena image is employed. The block size was eight for both techniques, and the reconstructed signal in each segment consisted of the two highest energy (unquantized) transform coefficients. The transform book contained $M = 8$ BT's, and in each segment of the signal, the best reference function yielding the least distortion was selected. As expected, DCT works adequately in the highly correlated regions, but performs worse especially in the regions with abrupt changes. Thus, the adaptive BT technique outperforms DCT both visually as well as in the mean square error sense. Three of the reference vectors, as generated via the LBG algorithm using various training signals, are displayed in Fig. 3(a).

2-D Case

The idea of adaptive BT can be easily extended to images. However, in this case the VQ algorithm is not run on the raw image data, but first a repertoire of image patterns generated by the outer product of 1-D vectors is obtained. The VQ algorithm is run on these 1-D vectors as detailed below.

- The training images are organized in normalized blocks of $N \times N$, denoted as $\boldsymbol{I}_l(i, j), l = 1, \cdots, L, i, j = 1, \cdots, N$ where N is typically 8.
- Image patterns are formed with the least square fitting of a square base $\boldsymbol{Y}_l$ to each image block. Here $\boldsymbol{Y}_l$ is obtained as the outer product of two row vectors, i.e., $\boldsymbol{Y}_l = \boldsymbol{r}_l^T \boldsymbol{c}_l, l = 1, \cdots, L,$ where $\boldsymbol{r}_l, \boldsymbol{c}_l \in \boldsymbol{R}^N$. Hence

$$||\boldsymbol{I}_l - \boldsymbol{Y}_l|| = ||\boldsymbol{I}_l - \boldsymbol{r}_l^T \boldsymbol{c}_l|| \tag{2}$$



(a) (b) (c)

Fig. 4. (a) Adaptive blocks have been superior to DCT on the indicated blocks at rate = 0.63 b/pixel. (b) Reconstructed image with DCT based coding. PSNR = 27.8 dB, 0.63 b/pixel. (c) Reconstructed image with hybrid VQ-OBT based coding. PSNR = 28.7 dB, 0.63 b/pixel.

is minimized where $\|\cdot\|$ denotes the Euclidean norm. These image patterns $(\boldsymbol{Y}_l)_{l=1,\cdots,L}$, form, in fact, a new training set for the VQ algorithm.

- As expected, this large set of image patterns is highly redundant. At this stage the VQ algorithm is applied to vectors $\{\boldsymbol{r}_l|l = 1,\cdots,L\} \cup \{\boldsymbol{c}_l|l = 1,\cdots,L\}$ to obtain a few cluster centers $\{\boldsymbol{h}_m|m = 1,\cdots,M\}$ that would represent as distinct image structures as possible.

An image block $\boldsymbol{I}$ is transformed as $\boldsymbol{A}_i\boldsymbol{I}\boldsymbol{A}_j$ where $\boldsymbol{A}_i$ and $\boldsymbol{A}_j$ are the transforms constructed from the selected $\boldsymbol{h}_i$ and $\boldsymbol{h}_j$'s. The selection is conducted by minimizing

$$\|\boldsymbol{I} - \boldsymbol{h}_p^T\boldsymbol{h}_q\|, \qquad p,q = 1,\cdots,M. \tag{3}$$

Several test images are used to evaluate the coding performance of the proposed technique. Simulation results indicate that the codebook sizes four to eight work quite well from both compaction performance and implementation complexity points of view. Fig. 3(b) demonstrates the representative M^2 image patterns formed from a codebook size of $M = 3$. Notice that these gray-level landscapes due to the $\{\boldsymbol{h}_i^T\boldsymbol{h}_j\}$ templates correspond to various gray-level facets.

In the flat regions of the image, the basis functions tend to be DC-like. For a DC reference vector, the BT design technique, PBT, leads to the Walsh–Hadamard transform (WHT) [10]–[12]. It is well known that the WHT does not work as well as DCT in image coding applications. Therefore, it should be argued that whenever an image block corresponds to a DC-like flat area, it should be coded with

TABLE I
8 × 8 OBT TYPE TRANSFORM HAVING DC FUNCTION AS A FIRST BASE FOR AR(1) INPUT SOURCE

$$\mathbf{A} = \begin{bmatrix} 0.3536 & 0.3536 & 0.3536 & 0.3536 & 0.3536 & 0.3536 & 0.3536 & 0.3536 \\ 0.4801 & 0.4212 & 0.2861 & 0.1011 & -0.1011 & -0.2861 & -0.4212 & -0.4801 \\ 0.4533 & 0.2009 & -0.1854 & -0.4688 & -0.4688 & -0.1854 & 0.2009 & 0.4533 \\ 0.4229 & -0.0852 & -0.4856 & -0.2794 & 0.2794 & 0.4856 & 0.0852 & -0.4229 \\ 0.3636 & -0.3493 & -0.3577 & 0.3434 & 0.3434 & -0.3577 & -0.3493 & 0.3636 \\ 0.2844 & -0.4882 & 0.0949 & 0.4144 & -0.4144 & -0.0949 & 0.4882 & -0.2844 \\ 0.1932 & -0.4611 & 0.4612 & -0.1932 & -0.1932 & 0.4612 & -0.4611 & 0.1932 \\ 0.0985 & -0.2775 & 0.4163 & -0.4899 & 0.4899 & -0.4163 & 0.2775 & -0.0985 \end{bmatrix}$$

DCT, while the other image blocks containing strong sloping patterns should be coded with the appropriate transforms from the book. This hybrid technique improves the performance of the compression in both detailed and flat areas. Fig. 4 illustrates the blocks within which VQ-based coder is chosen, while in the rest of the image blocks DCT coding is being used. In the sequel of this work, only the hybrid scheme will be employed.

III. DESIGN TECHNIQUE I: PERMUTATION-BASED ORTHOGONAL BLOCK TRANSFORM (VQ-PBT)

One method to generate an orthogonal BT matrix from a given reference vector is to use systematic permutations and sign changes on the elements of the reference vector [10]–[12]. Thus, while the first row of the transform matrix is constituted of the reference vector itself, the other rows are simply given by permutation and sign change operations on the elements of the reference vector.

Consider now the first row of a transform block, denoted as $\boldsymbol{h}_0$. The row vector $\boldsymbol{h}_0$ has N components, $\boldsymbol{h}_0 = [h_0(0)\ h_0(1)\ \cdots\ h_0(N-1)]$, while the other rows are denoted as $\boldsymbol{h}_i, i = 1, \cdots, N-1$. Furthermore, rows of the transform matrices form orthonormal sets, that is

$$\boldsymbol{h}_i \boldsymbol{h}_j^T = \delta_{i-j}. \tag{4}$$

The construction of an orthogonal matrix given a reference vector $\boldsymbol{h}_0$, based on the permutations and sign changes of its components is outlined below.

Define *permutation* functions

$$\beta_0(j), \beta_1(j), \cdots, \beta_{N-1}(j), \qquad j = 0, 1, \cdots, N-1$$

and *sign change* functions

$$\alpha_0(j), \alpha_1(j), \cdots, \alpha_{N-1}(j), \qquad j = 0, 1, \cdots, N-1$$
$$\alpha_i(j) \in \{-1, 1\}$$

where the subscript denotes the row number, and the argument denotes the component position in a row. Then $\boldsymbol{h}_i$ is defined as

$$h_i[j] = \alpha_i(j) h_0[\beta_i(j)] \qquad j = 1, \cdots, N-1.$$

TABLE II
G_{TC} VALUES FOR BOTH VQ-OBT AND DCT ON THE CAMERAMAN IMAGE. EACH CLASS VALUE CORRESPONDS TO A PATTERN AND THE COMPACTION PERFORMANCE IS COMPUTED OVER THE REGION MATCHING THAT PATTERN

Class	G_{TC}(VQ-OBT)	G_{TC}(DCT)
(1,1)	182.5	182.2
(1,2)	27.3	26.6
(1,3)	23.9	23.1
(2,1)	29.9	29.8
(2,2)	11.1	10.3
(2,3)	14.5	14.0
(3,1)	26.3	25.3
(3,2)	12.3	11.7
(3,3)	11.4	10.6

This definition says that the jth element of the ith row of the transform matrix is the same as the element of $\boldsymbol{h}_0$ on the $\beta_i(j)$th position except with a sign change given by $\alpha_i(j)$.

Such permutation and sign change operations result in pairwise orthogonality in (4) if only if they satisfy the relationship given [11]–[12] as

$$\begin{aligned} &\alpha_i(k)\alpha_j(k) + \alpha_i(t)\alpha_j(t) \\ &\quad = 2\delta_{i-j} \qquad 0 \le i, j, k \le N-1 \\ &\beta_i^{-1}(\beta_j(k)) \\ &\quad = \beta_j^{-1}(\beta_i(k)) \qquad 0 \le i, j, k \le N-1 \end{aligned} \tag{5}$$

where $t = \beta_i^{-1}(\beta_j(k))$.

As an example, the 8 × 8 transform matrix formed from the vector $[h(0) \cdots h(N-1)]$ is shown at the bottom of the page.

$$\boldsymbol{A} = \begin{bmatrix} h(0) & h(1) & h(2) & h(3) & h(4) & h(5) & h(6) & h(7) \\ -h(1) & h(0) & h(3) & -h(2) & h(5) & -h(4) & -h(7) & h(6) \\ -h(2) & -h(3) & h(0) & h(1) & h(6) & h(7) & -h(4) & -h(5) \\ -h(3) & h(2) & -h(1) & h(0) & h(7) & -h(6) & h(5) & -h(4) \\ -h(4) & -h(5) & -h(6) & -h(7) & h(0) & h(1) & h(2) & h(3) \\ -h(5) & h(4) & -h(7) & h(6) & -h(1) & h(0) & -h(3) & h(2) \\ -h(6) & h(7) & h(4) & -h(5) & -h(2) & h(3) & h(0) & -h(1) \\ -h(7) & -h(6) & h(5) & h(4) & -h(3) & -h(2) & h(1) & h(0) \end{bmatrix}$$

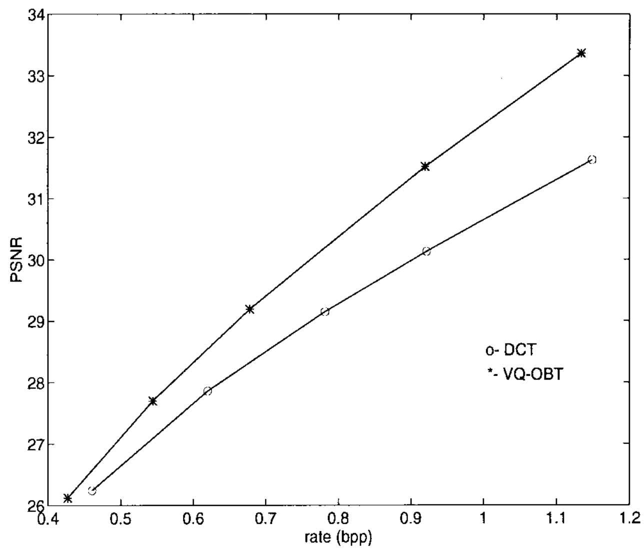


Fig. 5. PSNR versus rate curves for DCT-based and proposed adaptive coding method on the cameraman image.

This technique, a variation of Hadamard modulation of a reference vector, is in fact a generalization of the Walsh–Hadamard transform, since the lowpass function $\boldsymbol{h}$ will have in general components different than one's. The only restrictions on the reference vectors are their size, which must be an integer power of two.

In a permutation-based orthogonal block coder designed using the algorithm described above, all basis functions use the same set of coefficients (in different shift and sign positions). One can obtain linear phase BT's simply by imposing linear phase condition on $\boldsymbol{h}_i$'s. In other words, one can start with symmetric pattern vectors $\boldsymbol{r}_l = [\tilde{\boldsymbol{r}}_l \ \tilde{\boldsymbol{r}}_l \boldsymbol{J}], \boldsymbol{c}_l = [\tilde{c}_l \ \tilde{c}_l \boldsymbol{J}]$, where $\tilde{\boldsymbol{r}}_l, \tilde{c}_l$ are $N/2$ sized vectors and $\boldsymbol{J}$ is the counter identity matrix. We have found, however, that the resulting image pattern repertoire $\{\boldsymbol{Y}_l\}$ becomes quite restricted due to their horizontal and vertical double symmetry, and consequently the coding gains of VQ-PBT remain unimpressive with respect to that of DCT [13]. Thus, when the linear phase property is sacrificed, local structures are more frequently matched by these richer and more realistic image patterns. Notice that these patterns enable to code, e.g., diagonal edges, with a few coefficients while this would not be the case with symmetric bases.

IV. Design Technique II: Optimization-Based Block Transform (VQ-OBT)

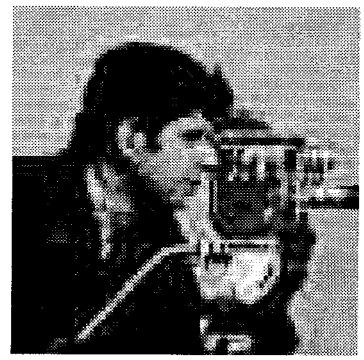
In the permutation-based transform design, as described above, only the first row (the so-called reference vector) was obtained from VQ so that it resembles the various image structures. Alternatively, it is possible to generate all the rows of the transform matrix out of an optimization scheme, thereby achieving an even higher compaction. This process leads to an orthogonal BT that no longer shares the same coefficient values. Again starting from $\boldsymbol{h}_0$'s as obtained from the VQ classification algorithm, one proceeds to select any $\boldsymbol{D}$ matrix of dimensions $(N-1 \times N)$, in the null space of $\boldsymbol{h}_0$ that is

$$\boldsymbol{D}\boldsymbol{h}_0 = \boldsymbol{0}$$

Since $\boldsymbol{D}\boldsymbol{D}^T > 0$ is a positive definite matrix, one can introduce a $\boldsymbol{Q}$ matrix such that

$$\boldsymbol{Q} = \boldsymbol{D}^T(\boldsymbol{D}\boldsymbol{D}^T)^{-(1/2)} \tag{6}$$

(a)



(b)

(c)

Fig. 6. (a) Original detail image. (b) Reconstructed detail image with DCT-based coding [detail of 4(b)]. (c) Reconstructed detail image with hybrid VQ-OBT based coding [detail of 4(c)].

and, thus, one can construct a transform matrix $\boldsymbol{Q}$ such that

$$\boldsymbol{A} = [\boldsymbol{h}_0 | \boldsymbol{D}^T(\boldsymbol{D}\boldsymbol{D}^T)^{-(1/2)}]^T. \tag{7}$$

The resulting matrix $\boldsymbol{A}$ is an orthonormal matrix satisfying, $\boldsymbol{A}\boldsymbol{A}^T = \boldsymbol{A}^T\boldsymbol{A} = \boldsymbol{I}_{N\times N}$ for any given reference vector $\boldsymbol{h}_0$. There are several ways to span the null space of $\boldsymbol{h}_0$, i.e., several possibilities for $\boldsymbol{D}$. Since our objective is to design an orthonormal transform having good compaction performance, we set an optimization procedure as follows.

The transform coefficient variances for a given input covariance matrix are found as the diagonal terms of the output autocorrelation matrix $\boldsymbol{R}_{yy}$ given by

$$\boldsymbol{R}_{yy} = \boldsymbol{A}\boldsymbol{R}_{xx}\boldsymbol{A}^T$$

where $\boldsymbol{R}_{xx}$ is the input autocorrelation matrix and $\boldsymbol{A}$ any orthogonal BT. Optimum compaction performance is obtained when geometric mean of the transform coefficient variances is minimum. Thus, we set the optimization problem as

$$\min\{J\} = \prod_{i=1}^{N} \mathrm{diag}_i\{\boldsymbol{A}\boldsymbol{R}_{xx}\boldsymbol{A}^T\} \tag{8}$$

with constraint

$$\boldsymbol{D}\boldsymbol{h}_0 = \boldsymbol{0}$$

for any given unit-norm $\boldsymbol{h}_0$ vector. Several BT coders are designed for reference vectors obtained from VQ algorithm.

To compare the performance of the VQ-OBT vis-a-vis that of their closest competitor, namely DCT, we assumed that the input statistics formed an AR(1) model with correlation coefficient ρ, as follows:

$$R_{xx}(r) = \rho^{|r|}.$$

This choice is motivated by the fact that this model is widely used for crude approximation of real images in literature. When we selected the reference vector $\boldsymbol{h}_0$ as the DC function, the resulting transform represents the optimum BT among all such transforms having the first row constant, which includes also the DCT. Its performance, as expected, remains in the corridor between those of KLT and DCT. Table I provides the coefficients of an OBT having a DC function as its first basis.

As expected, the coding gain performance of VQ-OBT is slightly better than that of VQ-PBT. This is due to the fact that VQ-OBT is more image class specific since all rows of the block transform are involved in the optimization search in contrast to VQ-PBT where only the first row is derived as such.

V. Performance Results

The performance results of the hybrid VQ-OBT scheme vis-a-vis those of a DCT-based algorithm are given in this section. The hybrid scheme opts for the DCT and Joint Photographers Expert Group (JPEG) quantization table in flat regions. On the other hand, in the more active regions, one of the $M^2 = 9$ bases is selected according to (3). Finally, the coefficients of the VQ-based transforms are treated with a (nonoptimized) uniform quantizer followed by an entropy coder.

Table II compares the energy compaction figures of VQ-OBT with those of the DCT scheme, where the class indication (i, j) refers to the waveforms in Fig. 3(a). In this table, G_{TC} denotes the transform coding gain [2]. Thus, for example, (2, 3) would indicate that in those blocks columns match "waveform 2" and that rows match "waveform 3" best; on the other hand (1, 1) would indicate "flat" blocks both row-wise and column-wise. One can observe that the energy compaction performance is better in every region where the algorithm has indicated the use of a specific non-DCT orthogonal transform.

The PSNR performance is also better over the whole image, e.g., for the cameraman image by margins of 0.5–1.5 dB in the 0.6–1 b/pixel range (Fig. 5). In this comparison, the entropy-coded header information has been included in the rate figures of the VQ-OBT technique. The cameraman image coded with hybrid VQ-OBT is shown in Fig. 4(c). For visual assessment of improvements, a detail from the face and camera region is shown in Fig. 6, where one can observe that the camera and its handle are both rendered more sharply.

VI. Conclusions

Two methods for signal adaptive orthogonal BT design have been advanced. Their adaptive nature is due to the fact that these BT's are generated from reduced size VQ bases. In the VQ-OBT technique, the autocorrelation sequence of the process must be estimated, but otherwise it leads to slightly better compression results. On the other hand, the VQ-PBT technique may be more practical in a wider range of applications and leads to multiplierless transformations [11].

The adaptive transforms have superior performance, especially in the highly textured and patterned regions, while the DCT bases remain superior in smooth regions. The proposed hybrid coder then tests each segment of the signal or block of the image and decides

- to use the DCT bases rather than the corresponding VQ-PBT (VQ-OBT) bases if the segment is DC-like;
- to use the corresponding VQ-PBT (VQ-OBT) bases if the segment is not DC-like, i.e., with high variance.

The proposed coder is found to have superior compaction performance as compared to DCT both in the rendition of details in the image and in the PSNR figures. The coder may form an alternative to DCT-based schemes in highly patterned classes of images.

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